PHYSICS 415 - Fall Semester 2025 - ODU

Nuclear and Particle Physics - Problem Set 1 - Solution

Problem 1)

^{6}Li is the only stable A=6 isobar, with Z=3 protons and N=3 neutrons (therefore with charge Q = 3e). Its natural abundance is 7.5% (the remainder is made up of ^{7}Li, the only other stable Li isotope). Its total angular momentum (nuclear spin) is J=1 and its ground state parity is positive (π=+1). Its "mass excess" (e.g. from the Website "Chart of Nuclides" listed on my Web page) is Δ = 14.086 MeV, which means that the neutral atom is heavier than 6u by this amount. This gives a total mass of 5603.05 MeV/c^2 for the neutral atom, and 5601.52 for the nucleus alone (after subtracting the electrons). This is 31.995 MeV/c^2 lighter than the combined mass of 3 protons and 3 neutrons. This means that the total binding energy is 31.995 MeV, while the average binding energy per nucleon is 5.333 MeV. Comparing the mass excess numbers with the neighboring nuclei ^{5}Li and ^{5}He, we find that the removal energy for a neutron is $\Delta(^5\text{Li}) + \Delta(n) - \Delta(^6\text{Li})$ = 5.665 MeV and that of a proton is $\Delta(^5\text{He}) + \Delta(^1\text{H}) - \Delta(^6\text{Li})$ = 4.593 MeV, not much different from the average binding energy of a nucleon. (Clearly, ^{6}Li is stable against proton or neutron decay).
Additional information can be found in the Nuclear Data tables, for instance the link "chart of isotopes" on my website. Here I find that the magnetic moment of ^{6}Li is about 0.822 nuclear magnetons, that it has a small quadrupole deformation of -0.818 mb, and that its nuclear radius is about 2 fm.

Problem 2)

a) From the mass formula, I get a predicted mass of 193,737.166 MeV/c^2. Using a table of mass excesses, I get 193,728.980 MeV/c^2. That means that the Lead nucleus is actually 8.186 MeV/c^2 "lighter" than predicted, or, conversely, more tightly bound by an extra binding energy of 8.186 MeV. This is not a huge discrepancy (no more than the average binding energy of a single nucleon), and it can be explained by the fact that ^{208}Pb is a "doubly magic" nucleus (i.e., both proton and neutron numbers are "magic", which means that they completely fill a shell, leading to a stronger binding).
b) In SI units, the energy of a charged sphere is $(3/20\pi\varepsilon_o)q^2/R$. Using the Ansatz $\rho_o = A/(4\pi R^3/3)$ for the density of nucleons in the nucleus, I conclude

$R=(3/4\pi\ \rho_o)^{1/3}A^{1/3} = 1.22$ fm $A^{1/3}$, or I can directly use $R_o = 1.2$ fm. Plugging it all in, I get $a_C = 0.708$ MeV/c^2. Pretty close to the value quoted in Povh et al.!

Problem 3)

Once again using mass excesses from the literature (or the Web), I get
$\Delta(^{40}K) = -33.535$ MeV (33.535 MeV lighter than 40/12 Carbon-12 atoms, i.e. more tightly bound); $\Delta(^{40}Ar) = -35.040$ MeV; $\Delta(^{40}Ca) = -34.846$ MeV.
So both potential daughter nuclei have a more negative mass excess than ^{40}K, which means their total masses are lower. This is due to the fact that ^{40}K is an odd-odd nucleus, which has a higher mass than the two neighboring even-even nuclei. As a result, ^{40}K should be able to decay into both of them. For the β^- decay into ^{40}Ca (Z increases), all we need is that $\Delta(^{40}K) > \Delta(^{40}Ca)$ which is the case. The maximum decay energy (mostly carried away by the electron and the antineutrino) is equal to the difference in mass excess, which is 1.311 MeV. The analog β^+ decay into ^{40}Ar (Z decreases) is **also** possible, since the difference is 1.505 MeV, which is sufficient to account for the two extra electron masses needed (1.022 MeV), one for the left-over "excess" electron that ^{40}K has more than ^{40}Ar and one for the positron created in the process. However, the remaining energy would be only 0.483 MeV (again, mostly carried away by the positron and the neutrino). If the decay proceeds via electron capture, the full 1.505 MeV can be transferred to the neutrino (and to atomic excitation of the daughter nucleus). In fact, all three decay modes really do occur in nature; β^- decay accounts for about 90% of all decays and β^+ decay plus electron capture for the remaining 10%. Surprisingly, the lifetime of ^{40}K is nevertheless very long - 1.3 Billion years! We will understand later that this is a consequence of the very high ground state spin of ^{40}K (4 units of h-bar), which means that all decays are strongly suppressed by selection rules. ^{40}K is famous because it is the lightest long-lived radioactive nuclide and responsible for a large fraction of the internal radiation dose received by all living things, due to the important biochemical role played by potassium

Problem 4)

The uranium is decaying with a half-life $t_{1/2}=4.5\cdot 10^9$ years. This corresponds to a decay constant $\lambda_U = \ln 2 / t_{1/2} = 4.88\cdot 10^{-18}$/s. On the other hand, the Thorium has a decay constant of $\lambda_{Th} = 3.329\cdot 10^{-7}$/s. The differential equation describing the creation and decay of Thorium is given by $dN_{Th}/dt = \lambda_U N_U - \lambda_{Th} N_{Th}$.
Since the ore was undisturbed, it is safe to assume that everything is in an equilibrium, i.e., $dN_{Th}/dt = 0$. Also, given its extremely long lifetime, we can assume N_U is roughly constant. This yields $N_{Th} = \lambda_U N_U / \lambda_{Th} = 1.46\cdot 10^{-11} N_U$. Using the mass number, I find that 1 kg of ^{238}U contains 4.2 mols, i.e. $2\cdot 53\cdot 10^{24}$ atoms. Therefore, there must be $3.71\cdot 10^{13}$ atoms of Thorium, or 0.0144μg.

Problem 5) (Extra Credit):

One could say that the mass of nuclei comes from the constituent nucleons (protons and neutrons) – in fact, nuclear masses are slightly smaller than the sum of the masses of all nucleons inside, as we discussed in class. You might think that this just continues to the next more fundamental level – nucleons in turn are made of quarks and gluons, so maybe their masses come from the mass of all the quarks and gluons inside combined? Unfortunately, that doesn't really work – gluons are actually massless, and the most prolific quarks inside nucleons (up and down quarks) have tiny masses – of order 10 MeV – compared to nucleon masses – nearly 1000 MeV. So, in fact, quark and gluon masses make up only a few % of nucleon masses – where does the rest come from? It turns out that there are 2 significant contributions – kinetic energy (quarks move around very fast inside nucleons) and the interaction energy between quarks and gluons (and gluons among themselves) – i.e., ultimately from the strange properties of QCD. You might ask "if the masses of nucleons are much larger than the masses of their constituents, why don't they decay into the latter – just like superheavy nuclei decay through fission or alpha decay?" This is indeed one of the most important puzzles about the strong force – and hence fundamental physics. Stay tuned…